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Seiberg duality in Chern-Simons theories with fundamental and adjoint matter

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Abstract: We explore the dynamics of three-dimensional Chern-Simons gauge theories with $\mathcal{N} = 2$ supersymmetry and matter in the fundamental and adjoint representations of the gauge group. Realizing the gauge theories of interest in a setup of threebranes and fivebranes in type IIB string theory we argue for a Seiberg duality that relates Chern-Simons theories with non-trivial superpotentials.

Keywords: Duality in Gauge Field Theories, Chern-Simons Theories.

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Contents

1. Introduction

There has been a recent resurgence of interest in Chern-Simons (CS) theories with varying amounts of supersymmetry. Chern-Simons theory without matter is a topological three dimensional field theory. It ceases to be topological, however, when it is coupled to matter. Then it exhibits non-trivial dynamics which are interesting for several reasons.

Conformal Chern-Simons theories with $\mathcal{N} = 8$ supersymmetry are expected to describe the low energy worldvolume dynamics of M2-branes in M-theory. Ref. [1] explored various Chern-Simons theories with this purpose, but did not find one with $\mathcal{N} = 8$ supersymmetry. Theories with this amount of supersymmetry were constructed in [2 – 6] and were related to M2-brane dynamics in [7, 8]. Theories with $\mathcal{N} = 6$ and $\mathcal{N} = 5$ supersymmetry were recently discussed in [9, 10]. A non-supersymmetric variant was considered in [11].

Via the AdS/CFT correspondence conformal Chern-Simons theories map to a class of four dimensional string/M theory backgrounds with negative cosmological constant. By studying them we may hope to learn more about string/M theory in these backgrounds.

Chern-Simons theories also arise in interesting condensed matter systems (see *e.g.* [12 – 16]). These include systems that exhibit quantum Hall effects or superconductivity. Supersymmetric Chern-Simons-matter theories are interesting in this respect as solvable toy models.

In this note we will focus on Chern-Simons-matter theories with $\mathcal{N} = 2$ supersymmetry (*i.e.* four real supercharges). These theories are characterized by a gauge group G, the Chern-Simons level k and the matter representations R_i [17]. For non-abelian gauge groups the level k is quantized. We will restrict to situations where the gauge group G is the unitary group $\mathrm{U}(N)$.

By adding superpotential interactions among the matter superfields one can break the conformal invariance and generate non-trivial renormalization group (RG) flows. In these situations, one would like to be able to determine the infrared (IR) behavior of the theory. In four dimensional gauge theories with four supercharges (*i.e.* $\mathcal{N} = 1$ supersymmetry

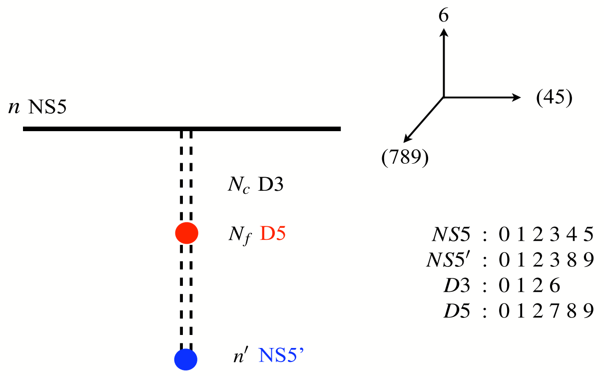

Figure 1: A configuration of D3, D5 and NS5-branes that realizes $\mathcal{N} = 2$ U(N_c) SQCD in three dimensions with two extra adjoint chiral superfields.

in 4d terms), there has been considerable progress in understanding such flows. Important tools in this progress are the NSVZ β-function formula [18], Seiberg duality [19], a-maximization [20, 21], *etc.* Similar progress in three dimensions would be desirable.

The recent reference [22] has taken a first step in this direction by proposing a Seiberg duality for $\mathcal{N} = 2$ Chern-Simons-matter theories with gauge group U(N_c) and N_f pairs of chiral multiplets $Q^i, \tilde{Q}_i$ $(i = 1, 2, \cdots, N_f)$. The superfields Q^i are in the fundamental representation of the gauge group and $\tilde{Q}_i$ in the anti-fundamental. A close cousin of this theory in four dimensions is $\mathcal{N} = 1$ SQCD. It is our purpose here to take a further step along these lines by studying $\mathcal{N} = 2$ Chern-Simons-matter theories with additional chiral multiplets in the adjoint representation. The analog of these theories in four dimensions is $\mathcal{N} = 1$ SQCD theories with adjoint chiral superfields [23 – 26]. We will postulate a Seiberg duality for these theories and provide some checks. We will focus mostly on the case of one adjoint chiral superfield. An interesting subtlety of a similar exercise with two adjoint chiral superfields will also be mentioned.

Section 2 formulates the theory of interest. We will find it convenient to phrase our statements in the language of brane configurations in type IIB string theory. Section 3 argues for a Seiberg duality and provides some checks. We conclude in section 4 with a summary of the main lessons and a list of interesting open problems.

2. The electric theory

We will realize the gauge theories of interest as low energy effective field theories residing in a configuration of threebranes and fivebranes in type IIB string theory in $\mathbb{R}^{9,1}$. This will provide a quick and intuitive access to many classical and quantum aspects of CS dynamics, which can be formulated, of course, independently in field theory language.

An instructive precursor of the configurations we want to consider appears in figure 1. This configuration preserves four supercharges, *i.e.* it exhibits $\mathcal{N}=2$ supersymmetry in the three directions (x^0, x^1, x^2) common to all the branes.[1] The low energy description of this system is in terms of a $\mathrm{U}(N_c)$ gauge theory that lives on the D3-branes which are suspended along the 6-direction between the n NS5-branes and the n' NS5$'$-branes. The matter content of this theory comprises of: (a) an $\mathcal{N}=2$ vector multiplet V, (b) N_f pairs of $\mathcal{N}=2$ chiral multiplets Q^i, $\tilde{Q}_i$ $(i=1,2,\cdots,N_f)$ in the fundamental and anti-fundamental representations of the gauge group and (c) two chiral supermultiplets X, Y in the adjoint representation.

The vector multiplet V arises from 3-3 strings on the D3-branes and includes both the gauge field A_μ $(\mu=0,1,2)$ and a scalar σ. The scalar parametrizes the position of the D3-branes in the x^3 direction. The N_f pairs of chiral multiplets Q, $\tilde{Q}$ arise from the 3-5 strings stretching between the D3 and the D5-branes. Finally, the chiral multiplets X, Y arise from 3-3 strings and their scalar components parametrize the position of the D3-branes along the (89) and (45) directions respectively.

The dynamics of the vector multiplet V and chiral multiplets $Q, \tilde{Q}$ are those of $\mathcal{N}=2$ SQCD in three dimensions. The extra adjoint chiral superfields have a non-trivial superpotential which fixes the values of their scalar components. We will not discuss further the dynamics of this system since it lies outside our scope. Instead, we will now proceed to consider a closely related theory that arises from that of figure 1 by a certain deformation.

Let us only note in passing that by compactifying the direction x^3 and T-dualizing the configuration of figure 1 we obtain in type IIA string theory a configuration that realizes at low energies $\mathcal{N}=1$ SQCD in four dimensions with two adjoint chiral superfields [28, 29]. This theory has a polynomial superpotential of the form

$$W = \frac{s_0}{n+1}\,\mathrm{Tr}\,X^{n+1} + \frac{s_0'}{n'+1}\,\mathrm{Tr}\,Y^{n'+1} + \mathrm{Tr}[X,Y]^2 + \tilde{Q}_i Y Q^i \ . \tag{2.1}$$

In the brane configuration s_0, s_0' should be thought of as large numbers.

The deformation we want to consider is the following. Start with the configuration in figure 1 with $N_f + k$ D5-branes. Then, move k of these D5-branes along the x^6 direction until they intersect with the n' NS5$'$-branes. At this point we can locally reconnect the D5 and NS5$'$ branes as in figure 2 to obtain an (n', k) fivebrane bound state. The resulting configuration will continue to preserve the same amount of supersymmetry provided that the (n', k) fivebrane is rotated in the (37) plane with a specific angle θ. This angle is determined by the integers n', k via the relation [30]

$$\tan\theta = g_s \frac{k}{n'} \ . \tag{2.2}$$

After the reconnection we send the NS5$'$ and D5-branes to infinity to be left with the configuration of figure 3 whose dynamics are described at low energies by the theory we are interested in.

[1] With both bunches of the NS5-branes parallel along (012345), instead of orthogonal, we would obtain $\mathcal{N}=4$ supersymmetry. That configuration, with $n=n'=1$, is the one analyzed in the original work [27].

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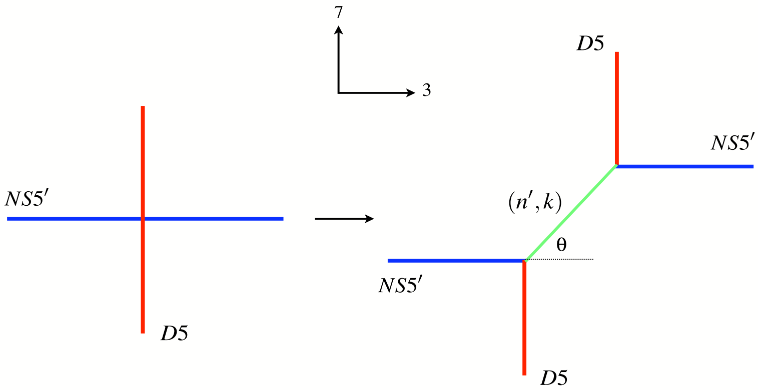

Figure 2: k D5-branes recombining with n' NS5-branes to give an (n', k) fivebrane bound state.

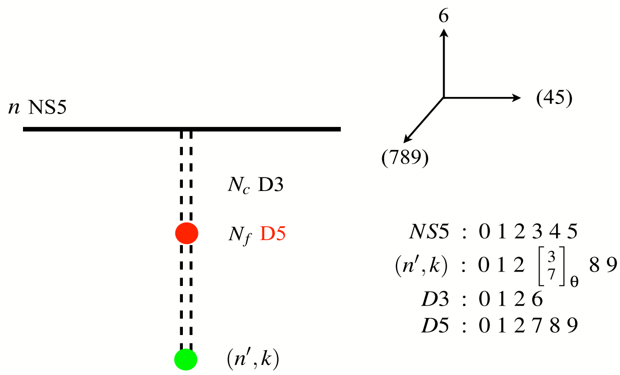

Figure 3: The configuration of branes after the reconnection of k D5-branes and n' NS5′-branes. The notation $\left[\begin{smallmatrix}3\\7\end{smallmatrix}\right]_\theta$ denotes the fact that the (n', k) brane is rotated in the (37) plane with an angle θ.

The low energy theory on the D3-branes suspended between the NS5 and (n', k) branes still has $\mathcal{N} = 2$ supersymmetry and the low energy degrees of freedom are again given by the vector multiplet V, the N_f pairs of chiral multiplets Q^i, $\tilde{Q}_i$ and the adjoint chiral multiplets X, Y. However, the dynamics of these fields are now different.

Most notably, the rotation of the (n', k) fivebrane in the (37) plane suggests that the scalar σ has become massive. By supersymmetry the whole vector multiplet must become massive. In three dimensions a vector field can become massive without spoiling gauge invariance by adding to the Lagrangian the Chern-Simons interactions. Indeed, it was argued in [31, 32] that the gauge theory on D-branes ending on (p, q) fivebrane bound states includes the Chern-Simons Lagrangian at fractional level $\frac{q}{p}$. A fractional level is acceptable for a $\mathrm{U}(1)$ gauge theory, but it is in conflict with the level quantization in non-abelian gauge theories. For a recent discussion on this point see [33]. In order to avoid this issue, in the rest of this note we will restrict attention to the special case where $n' = 1$. In that case, our configurations realize a Chern-Simons theory with integer level k.

The dynamics of the remaining multiplets are unaffected by the deformation. Hence, we end up at low energies with an $\mathcal{N} = 2$ Chern-Simons theory at level k coupled to N_f pairs of chiral multiplets $Q^i, \tilde{Q}_i$ and two adjoint chiral superfields X, Y. For reasons that will become apparent momentarily the theory also possesses a superpotential of the form (2.1). For $n' = 1$ the superfield Y is massive and can be integrated out. Then we are left with a single adjoint superfield, the superfield X, that has the superpotential

$$W = \frac{s_0}{n+1} \operatorname{Tr} X^{n+1} \ . \tag{2.3}$$

As a first check, notice that for $n = 1$ the superfield X is also massive and can be integrated out. Then, as anticipated, we recover the theory of ref. [22].

In the brane configuration of figure 3 (for $n' = 1$) there are $N_c - nN_f$ D3-branes that have to stretch between the n NS5-branes and the $(1, k)$ bound state. According to the s-rule of ref. [27] supersymmetry is preserved if the maximum of these branes is nk, hence the constraint

$$nN_f + nk - N_c \geq 0 \ . \tag{2.4}$$

In our $\mathcal{N} = 2$ CS theory this is a necessary property for the existence of a supersymmetric vacuum. It is worth comparing this condition to a corresponding condition for stability in the four dimensional $\mathcal{N} = 1$ SQCD with a single adjoint chiral superfield. In that case the condition is $nN_f - N_c \geq 0$ [24].

The superpotential (2.3) can be deduced from the brane moduli space in the following way. Displace the n NS5-branes in the (89) plane and place them at n different points $a_j = x_j^8 + ix_j^9$, $j = 1, 2, \cdots, n$. Then, the N_c D3-branes will also break up into n groups of r_1 D3-branes ending on the a_1 positioned NS5-brane, r_2 D3-branes ending on the a_2 positioned NS5-brane *etc.* with

$$\sum_{i=1}^{n} r_i = N_c \ . \tag{2.5}$$

From the D3-brane point of view a_i are the expectation values of the diagonal matrix elements of the complex scalar in the superfield X. In order to account for these vacua in gauge theory a polynomial superpotential is needed of the form

$$W(X) = \sum_{i=0}^{n} \frac{s_j}{n+1-i} X^{n+1-i} \ . \tag{2.6}$$

For generic coefficients $\{s_j\}$ the superpotential has n distinct minima $\{a_j\}$ related to $\{s_j\}$ via the relation

$$W'(x) = \sum_{i=0}^{n} s_i x^{n-j} = s_0 \prod_{i=1}^{n} (x - a_i) \ . \tag{2.7}$$

In the gauge theory picture the integers $(r_1, \cdots, r_n)$ label the number of the eigenvalues of the $N_c \times N_c$ matrix X residing in the ith minimum (for r_i) of the potential $V = |W'(x)|^2$. When all the expectation values a_j are distinct the adjoint field is massive and the gauge group is Higgsed

$$\mathrm{U}(N_c) \to \mathrm{U}(r_1) \times \mathrm{U}(r_2) \times \cdots \times \mathrm{U}(r_n) \ . \tag{2.8}$$

In this vacuum we get n decoupled copies of the $\mathcal{N} = 2$ CS theories with fundamentals that were considered in [22].

The superpotential (2.3) is a classically relevant interaction for $n = 1, 2$. For $n = 1$ we recover in the IR the theory of [22]. For $n = 2$ we are driven towards a different IR theory whose precise properties are unknown. For $n = 3$ the interaction is classically marginal. It has been argued in [17] that this interaction drives the theory towards an interacting IR fixed point. Finally, for $n \geq 4$ the interaction is classically irrelevant. In analogy to the four dimensional case of $\mathcal{N} = 1$ adjoint-SQCD, we would like to propose that these interactions are in fact *dangerously irrelevant*. The corresponding operators develop large anomalous dimensions in the theory without the superpotential interaction and become relevant, hence they can affect the IR physics in a non-trivial manner when added to the Lagrangian. Unfortunately, the necessary technology is currently lacking to verify this statement explicitly.

The global symmetry of the theory is

$$\mathrm{SU}(N_f)_v \times \mathrm{SU}(N_f)_a \times \mathrm{U}(1)_a \times \mathrm{U}(1)_R \ . \tag{2.9}$$

The first three of these symmetries become obvious in the brane setup by moving the N_f D5-branes along x^6 on top of the $(1, k)$ fivebrane and performing separate $\mathrm{U}(N_f)$ transformation on the portions of the D5s with $x^7 > 0$ and $x^7 < 0$. The last one is an R-symmetry. The theory has two R-symmetries, but only under one of them is the superpotential (2.3) invariant. In the brane setup these symmetries are related to the geometric rotation symmetries $\mathrm{U}(1)_{45}$, $\mathrm{U}(1)_{89}$ along the (45) and (89) planes respectively.

Other deformations of the field theory involving the quark superfields Q^i, $\tilde{Q}_i$ are also easy to see in the brane picture. For example, moving the D5-branes in the (45) plane corresponds in field theory to turning on complex masses for $Q, \tilde{Q}$ via the superpotential interaction $W = m_i \tilde{Q}_i Q^i$. Moving the D5s in the x^3 direction corresponds to turning on real masses with opposite signs to Q, $\tilde{Q}$.

The field theory has a large moduli space $\mathcal{M}$ parametrized by the expectation values of the scalar components of the quark superfields Q^i, $\tilde{Q}_i$. This space arises in the brane construction by separating the N_f D5-branes in the x^6 direction and then splitting the D3-branes on them [28, 29]. The complex dimension of the moduli space is

$$\dim \mathcal{M} = \begin{cases} nN_f^2 \, , & N_f \leq m \\ 2N_f N_c - nm^2 - p(2m+1) \, , & N_f > m \end{cases} \tag{2.10}$$

where we decomposed the number of colors N_c as

$$N_c = nm + p \, , \quad m, \ p \in \mathbb{Z}_{\geq 0} \, , \quad 0 \leq p < n \ . \tag{2.11}$$

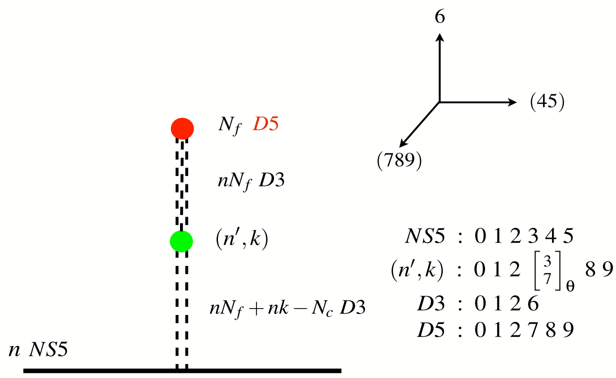

Figure 4: The magnetic configuration of branes for general n, n'.

3. The magnetic theory

By moving the D5-branes and the $(1, k)$ fivebrane past the n NS5-branes along the x^6 direction we obtain, as in [28, 29], the configuration that appears in figure 4. When a (p, q)5-brane passes through a (p', q')5-brane $|qp' - pq'|$ D3-branes are created [31]. Hence, in figure 4 (for $n' = 1$) nN_f D3s are stretched between the D5s and the $(1, k)$ fivebrane and $nN_f + nk - N_c$ D3s are stretched between the $(1, k)$ fivebrane and the n NS5s.

Assuming that the infrared dynamics are not affected by this process we end up with a gauge theory which is Seiberg dual to the original. The dual theory lives on the D3-branes stretching between the $(1, k)$ fivebrane and the n NS5-branes. It is $\mathcal{N} = 2$ CS at level k with gauge group $\mathrm{U}(nN_f + nk - N_c)$, N_f pairs of chiral multiplets $q_i, \tilde{q}^i$, an adjoint chiral superfield Y and n magnetic mesons M_i $(i = 1, \cdots, n)$, each of which is an $N_f \times N_f$ matrix. The magnetic mesons arise from 3-3 strings residing on the nN_f D3-branes stretching between the D5s and the $(1, k)$ fivebrane.[2] As in [28], a superpotential of the form

$$W = -\frac{s_0}{n+1} \operatorname{Tr} Y^{n+1} + \sum_{i=1}^{n} M_i \tilde{q} Y^{n-i} q \tag{3.1}$$

is anticipated. We notice that the rank of the dual gauge group is $\tilde{N}_c = nN_f + nk - N_c$. The positivity of this rank is equivalent to the condition (2.4) for the existence of a supersymmetric vacuum in the electric theory.

A potential concern for the validity of Seiberg duality in this system stems from the fact that in the above transformation there is a singularity when the NS5-branes meet with the $(1, k)$ fivebrane during their motion along x^6. In the case of NS5/NS5$'$ configurations

[2] A naive counting of the Chan-Paton indices for 3-3 strings appears to give $nN_f \times nN_f$ massless fields at the origin of moduli space. It has been pointed out, however, in a related context in [28] that this counting is misleading (see section 7.3 of [28]).

as in [28], this singularity can be avoided by separating the NS5 and NS5′-branes along their common transverse direction x^7. This is not possible, however, in our configuration, since the $(1,k)$ fivebrane is rotated with some non-zero angle along the (37) plane. With this point noted, let us accept as a working assumption that the CS theories of this and the previous section are indeed dual to each other and see if we can make any checks of this tentative duality.

Repeating the arguments of ref. [22] one can show that the proposed duality is consistent with the structure of the moduli space and deformations. The magnetic configuration has the same global symmetries (2.9) as the electric configuration. This can be seen directly in the brane setup as in the previous section.

The moduli space of the magnetic configuration arises by separating the D5-branes along the x^6 directions and then splitting the nN_f D3-branes stretched between them and the $(1,k)$ fivebrane consistently with the geometry [29]. Counting the dimension of the resulting moduli space one recovers the expressions of the electric case (2.10). As in ref. [22] it is important to notice in this exercise that when $N_f > m$ the $n(N_f+k)-N_c$ D3-branes stretching between the $(1,k)$ fivebrane and the n NS5-branes are more than nk contrary to the s-rule. Then, to preserve supersymmetry one has to keep $nN_f - N_c$ flavor D3-branes at the origin. With this restriction one recovers the second expression of eq. (2.10). The agreement between the moduli spaces of the electric and magnetic theories can also be deduced easily in the case of the general deformation (2.6) and the associated Higgsing (2.8). There is a corresponding deformation of the magnetic theory in this case with a superpotential

$$W_{\text{magn}} = -\sum_{i=0}^{n} \frac{\bar{s}_i}{n+1-i} \operatorname{Tr} Y^{n+1-i} + \sum_{i=1}^{n} \bar{\bar{s}}_i M_i \tilde{q} Y^{n-i} q \tag{3.2}$$

where $\bar{s}_i = \bar{s}_i(\{s_j\})$, $\bar{\bar{s}}_i = \bar{\bar{s}}_i(\{s_j\})$ are functions of s_j whose precise form can be deduced with the methods of [25]. The equality of the dimensions of the moduli spaces of the electric and magnetic descriptions for each copy of the decoupled $\mathcal{N}=2$ CS theories was checked in [22].

Several deformations of the electric theory can be matched directly to the magnetic theory precisely as in [22]. Since the analysis presented there can be repeated here *mutatis mutandis* we will not be explicit. As an example, we note that deforming the electric theory by the superpotential $W = m_1 \tilde{Q}_1 Q^1$ corresponds in figure 3 to separating one of the N_f D5-branes in the (45) plane from the D3-branes. In the magnetic description this deformation requires one of the D3-branes connected to the D5-branes to combine with one of the $nN_f + nk - N_c$ D3s thus reducing the gauge group by one.

4. Closing remarks

In this note we considered the possibility of Seiberg duality in Chern-Simons theories with $\mathcal{N}=2$ supersymmetry and matter in the fundamental and adjoint representations. Generalizing the arguments of [22] to include a field in the adjoint representation we found evidence for a duality between $\mathrm{U}(N_c)$ and $\mathrm{U}(nN_f+nk-N_c)$ CS theories both at integer level

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k. N_f is the number of flavor chiral multiplets $Q^i, \tilde{Q}_i$. The U(1) part of the gauge groups is interacting and important in these theories. This tentative duality is a strong/weak coupling duality in the sense that when $k \to \infty$, with N_c, N_f fixed, the electric description becomes weakly coupled, whereas the magnetic description becomes strongly coupled [22].

There are several aspects of this work that deserve further study. For instance, the above theories contain a superpotential interaction $W \propto \mathrm{Tr}\, X^{n+1}$ by an operator that is classically irrelevant when $n > 3$. We proposed that this is a dangerously irrelevant operator. It would be interesting to verify this explicitly.

In this note we did not discuss extensively the case of two adjoint chiral superfields. As we mentioned in the main text, this case can be achieved by considering the general n, n' brane setups of figures 3, 4. What needs to be understood better is the CS theory that arises in this system at low energies. Naively, this is a theory with fractional CS level. This is perfectly consistent for U(1) gauge groups, however, it is inconsistent with the quantization of the level for non-abelian gauge groups. It has been proposed that in this case extra interactions need to be taken into account [32, 33].

We hope to return to some of these issues in a future publication.

Acknowledgments

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